Proceedings – 23rd Annual Conference – IEEE/EMBS Oct.25-28, 2001, Istanbul, TURKEY

TIKHONOV REGULARIZATION USING A MINIMUM-PRODUCT CRITERION: APPLICATION TO BRAIN ELECTRICAL TOMOGRAPHY

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Abstract – **Tikhonov regularization is applied to the inversion of EEG potentials. The discrete model of the inversion problem results from an analytic technique providing information about extended intracranial distributions, with separate current source and sink positions. A three-layered concentric sphere model is used for representing head geometry. The selected regularization parameter is the minimizer of the product of the norm of the Tikhonov regularized solution and the norm of the corresponding residual. The simulations performed indicate that this regularization parameter selection method is more robust than the empirical Composite REsidual and Smoothing Operator approach, in cases where only gaussian measurement noise exists in the discrete inverse model equation. Therefore the minimum product criterion can be used in real Evoked Potentials' data inversions, for the creation of brain electrical activity tomographic images, when the amount of noise present in the measured data is unknown.**

Keywords **– EEG Inverse problem, Evoked Potentials, Tikhonov Regularization, Minimum-Product Criterion**

I. INTRODUCTION

Discrete ill-posed problems arise in many natural sciences applications [1]. A discrete linear model equation of the following form has to be solved:

$$\mathbf{A}\cdot\underline{X} = \underline{Y} \tag{1}$$

where K is a mxn matrix. The problem is ill posed in the sense that small variation in the data matrix $\underline{Y}$ may lead to arbitrarily large changes in the solution. This is reflected in the ill conditioning of K. If (1) is solved using the singular value decomposition (SVD) method, then the contribution of the small singular values will result in magnifying the noise present in $\underline{Y}$. Therefore a regularization of the problem is required in order to filter out the influence of the noise.

Tikhonov regularization is commonly used in inverse electromagnetic problems. The regularization parameter, which controls the amount of filtering introduced by regularization, is selected using appropriate criteria such as the Composite REsidual and Smoothing Operator (CRESO) [2], the Generalized Cross-Validation (GCV) [3] and the L-curve [4-6]. Recently a minimum-product approach has been proposed for the selection of the regularization parameter [7], closely related to the L-curve approach. It is based on the minimization of the product of the norm of the regularized solution and the norm of the corresponding residual. Applications have been presented in the fields of electrocardiography and electroencephalography [8-10].

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In previous studies a novel Brain Electrical Tomography method was presented, for solving the EEG inverse problem, primarily used in inverting late low-frequency Evoked Potentials to current distribution maps inside the brain [11-15]. The method provides information about extended current source distributions in widespread brain regions. This characteristic of the method is due to the electric field analysis used, which enables the observation of separated and distant current sources and sinks. The analytical equations of the inverse electromagnetic problem are stated using Green's functions and then discretized.

The present work aimed at studying the quality of inverse solutions provided by the Brain Electrical Tomography method, based on simulated head surface potentials using the minimum-product approach for selecting the Tikhonov regularization parameter, in comparison to the CRESO approach.

II. METHODOLOGY

The forward problem has been previously solved through an analytical formulation that enables the differentiation of intracranial positive and negative current source activity, using a three-layered concentric spherical head model [11]. Simulated head potentials Vol_i, resulting from a known initial distribution $\underline{X}_{INIT}$, are computed at m finite points $\underline{r}_i$, i=1,...,m. For solving the inverse problem the brain region supposed active is divided into n small voxels Δu_k ,with centres at $\underline{r}_k$, carrying an unknown average volume source current density ρ_{J_K} , k=1,...,n. The inversion problem equation can be stated in matrix form as in (1), where $\underline{Y}=[Vol_i]$, i=1,...,m, $\underline{X}=[x_k]$, $x_k=\rho_{J_K}\,\Delta u_k$, k=1,...,n, $\mathbf{A}=[a_{ik}]$, $a_{ik}=G_3(\underline{r}_i,\underline{r}_k)$ and G_3 is the known Green function of the problem.

In the zero-order Tikhonov regularization the following functional is minimized [16]:

$$M_t(\underline{X}) = \|\mathbf{A}\cdot\underline{X}-\underline{Y}\|^2+t\|\underline{X}\|^2 \tag{2}$$

The regularized solution to (1) is given by:

$$\underline{X}(t) = (\mathbf{A}^T\cdot\mathbf{A}+t\mathrm{I})^{-1}\cdot\mathbf{A}^T\cdot\underline{Y} \tag{3}$$

In order to find the optimum value t_{OPT} , which gives the solution $\underline{X}(t)$ closest to $\underline{X}_{INIT}$, we used, according to the minimum product criterion, the value t_P that minimizes the product:

Report Documentation Page

Report Date	Report Type	Dates Covered (from... to)
25 Oct 2001	N/A	-

Title and Subtitle Tikhonov Regularization Using a Minimum-Product Criterion: Application to Brain Electrical Tomography	**Contract Number**
	Grant Number
	Program Element Number
Author(s)	**Project Number**
	Task Number
	Work Unit Number
Performing Organization Name(s) and Address(es) Department of Medical Instrumentation Technology Technological Education Inst of Athens Athens, Greece	**Performing Organization Report Number**
Sponsoring/Monitoring Agency Name(s) and Address(es) US Army Research, Development & Standardization Group (UK) PSC 802 Box 15 FPO AE 09499-1500	**Sponsor/Monitor's Acronym(s)**
	Sponsor/Monitor's Report Number(s)

Distribution/Availability Statement
Approved for public release, distribution unlimited

Supplementary Notes
Papers from 23rd Annual International Conference of the IEEE Engineering in Medicine and Biology Society, October 25-28, 2001, held in Istanbul, Turkey. See also ADM001351 for entire conference on cd-rom.

Abstract

Subject Terms

Report Classification unclassified	**Classification of this page** unclassified
Classification of Abstract unclassified	**Limitation of Abstract** UU

Number of Pages
4

$$\mathrm{P}(t) = \|\underline{\mathrm{X}}\|\|\mathbf{A}\cdot\underline{\mathrm{X}}-\underline{\mathrm{Y}}\| \quad (4)$$

Alternatively the CRESO regularization parameter t_{CRE} might be used, determined as the smallest value of t>0 that results in a local maximum of the function:

$$\mathrm{C}(t) = \|\underline{\mathrm{X}}(t)\|^2 + 2t\frac{d}{dt}\|\underline{\mathrm{X}}(t)\|^2 \quad (5)$$

The performance of the inversion technique is checked by computing a set of critical parameters, including the relative error $\mathrm{RE}(t)=\|\underline{\mathrm{X}}(t)-\underline{\mathrm{X}}_{INIT}\|/\|\underline{\mathrm{X}}_{INIT}\|$, the residual $\mathrm{RD}(t)=\|\mathbf{A}\cdot\underline{\mathrm{X}}-\underline{\mathrm{Y}}\|$ and the "mean absolute percent potential error" $\mathrm{ME}(t)=\{\sum_{i=1}^{m}|(y_i-<\underline{a}_i,\underline{\mathrm{X}}(t)>)\mathrm{x}100/y_i|\}/\mathrm{m}$, where $\underline{a}_i=(a_{i1},...,a_{in})$ is the i_{th} line of matrix **A** and $<\underline{a}_i,\underline{\mathrm{X}}(t)>$ is the inner product of vectors $\underline{a}_i$ and $\underline{\mathrm{X}}(t)$. Parameters RD and ME reflect the accuracy by which the algorithm solves (1).

III. Results and Discussion

The model used in the inversion process had shell radii r_1=8.8cm, r_2=9.7cm, r_3=10.0cm, shell conductances $\sigma_1=\sigma_3$=0.3S/m, σ_2=0.0042S/m. It was m=120, n=60 and the condition number for matrix **A** was 1115. The investigated source region was restricted to a two-dimensional spherical shell described by $0<\theta<360^0$, $0<\varphi<80^0$ and r_s=8.3cm, corresponding to the outer layer of the cortex. Since both sources and sinks are positioned in this shell the investigated current flow is tangential to the spherical border surface, reflecting mainly sulcal cortical activity.

In Table I we present the values of the performance parameters for the Tikhonov regularization technique, for $t=t_{OPT}$ and $t=t_P$, with $\underline{\mathrm{X}}_{INIT}$ given in Fig.1(a), when increasing amounts of gaussian noise were added to the noiseless voltage data $\mathbf{A}\cdot\underline{\mathrm{X}}_{INIT}$. In Fig.2 we present the plots of RE(t) and P(t), for the various noise levels. As can be seen, the minimum-product criterion provides a solution $\underline{\mathrm{X}}(t_P)$, which reconstructs $\underline{\mathrm{X}}_{INIT}$ similarly to the reconstruction provided by the optimum Tikhonov solution $\underline{\mathrm{X}}(t_{OPT})$, as reflected by the relation between RE(t_P) and RE(t_{OPT})=min{RE(t)}. It should be noted that the CRESO criterion failed to provide a value t_{CRE}, since the function C(t) did not present local maxima.

In the present work there was no correlated noise in (1). It is already known that if only correlated noise exists in the discrete model equation, then most of the methods used for locating an appropriate $t\approx t_{OPT}$, such as the CRESO criterion, GCV, L-curve corner, as well as the minimum-product criterion fail [5,6,10]. The main finding of the inverse problem simulations performed in the present work is that the minimum-product criterion tends to be more robust in detecting a regularization parameter than the CRESO criterion, even when only gaussian noise is present

Table I
Inversion Technique Performance Parameters for Simulated EEG Data

Noise level (%)		RE	RD (nV)	ME
0.5	$t=t_{OPT}=3 \times 10^{-2}$	0.038	9.3	0.36
	$t=t_P=2.5 \times 10^{-3}$	0.040	9.0	0.37
1	$t=t_{OPT}=4.5 \times 10^{-3}$	0.063	16.9	0.53
	$t=t_P=9 \times 10^{-3}$	0.066	16.5	0.52
5	$t=t_{OPT}=6.5 \times 10^{-1}$	0.282	98.1	4.21
	$t=t_P=3 \times 10^{-1}$	0.300	91.2	3.99
10	$t=t_{OPT}=1.5 \times 10^{0}$	0.412	199.8	8.60
	$t=t_P=3 \times 10^{0}$	0.440	220.7	9.20

in the data. Taking into account that Algebraic Reconstruction Techniques (ART) algorithms, which are an alternative method for solving (1), require at least an approximate knowledge of the noise level present in the measurements [15], the application of the minimum-product criterion enhances the probability to create reliable brain electrical activity tomographic images, when the amount of noise present in the measured data is unknown.

A final cautionary remark has to be made concerning a problem existing in all the reconstructions tested in this work, apparent also from Fig.1(b), i.e. that the reconstructed distributions tend to be smeared and extended replicas of the initial distribution. This is a well-known phenomenon in the inverse EEG problem and is due to the use of L_2-norm solution algorithms [17]. In cases when the underlying cortical electrical activity is suspected to present a highly "spiky" profile, then the distribution to reconstruct may not be recovered using any of the L_2-norm solution methods, but may instead require the use of L_1-norm methods. This stresses the utility of any prior available information on brain structure and function, in order to discard mathematical solutions, which do not reflect real brain phenomena.

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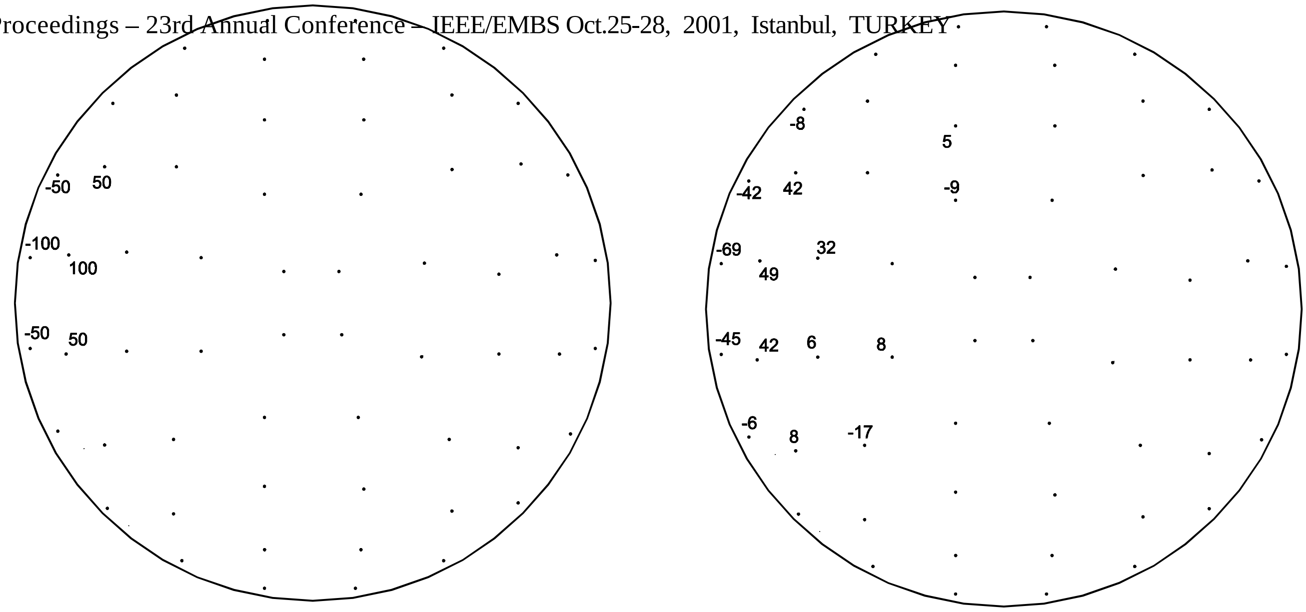

(a) (b)

Fig. 1. Brain electrical activity on a two-dimensional spherical layer ($0< \varphi <360^0$, $0< \vartheta <60^0$) positioned at r=8.7 cm, in nanoAmperes. Current source amplitudes smaller than 5 nanoAmperes were omitted for clarity. (a) Initial current distribution $\underline{X}_{INIT}$, (b) Tikhonov solution using the *t* value indicated by the minimum product criterion, i.e. $\underline{X}(t_p)$. 10% gaussian noise was added to the simulated potential data. Each dot represents the planar projection of the center of a voxel of the investigated brain region.

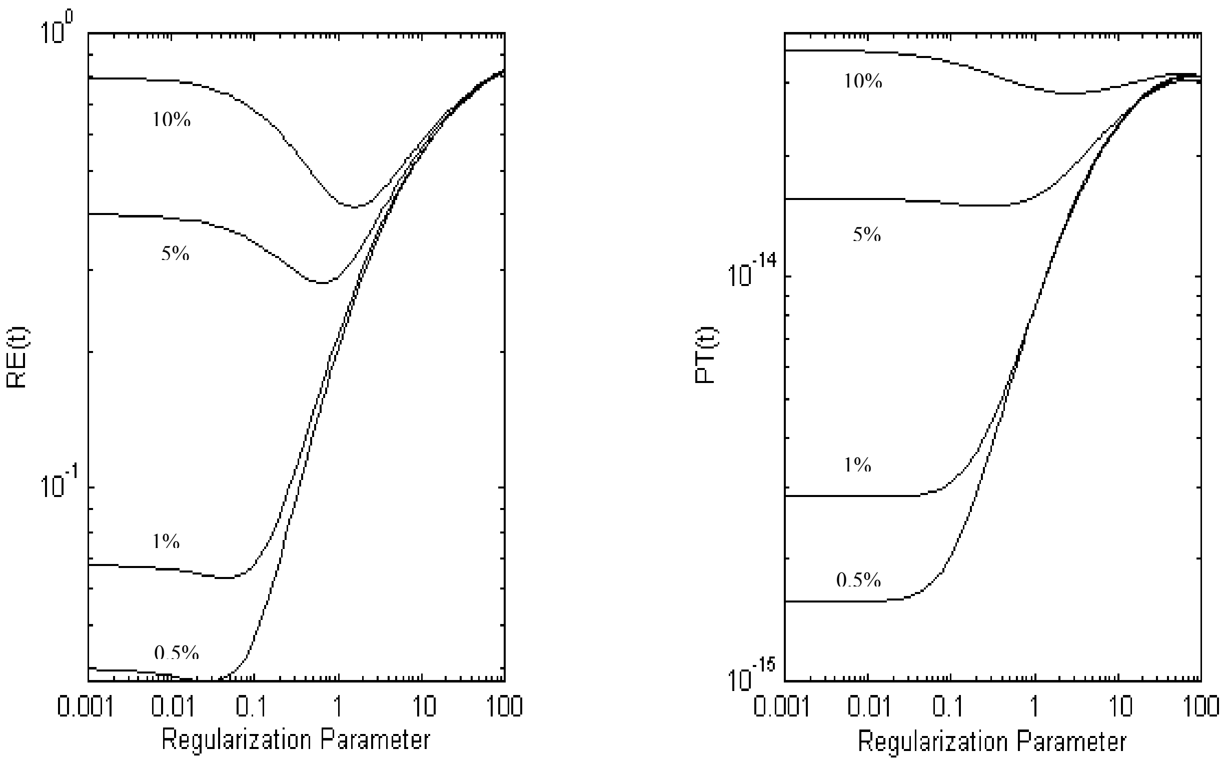

Fig. 2. Evolution of the relative error RE (left) and the product PT (right) as a function of the regularization parameter *t*, for the various gaussian noise levels. PT is expressed in Volts^2.

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